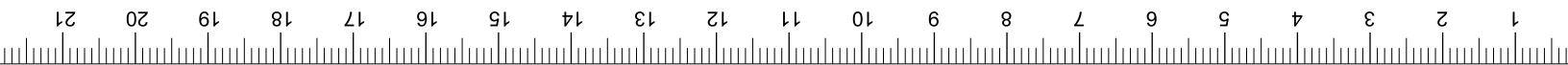# Helping Children Learn Mathematics

Robert Reys | Mary Lindquist | Diana Lambdin | Nancy Smith

11 EDITION

11TH EDITION

HELPING CHILDREN LEARN MATHEMATICS
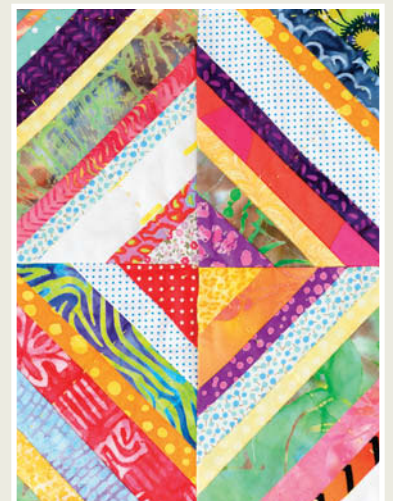
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Preface

Welcome to the 11th edition of *Helping Children Learn Mathematics*. Earlier editions have helped several generations of elementary teachers. We are grateful to the many instructors and students who have written us with ideas, stories, and suggestions about ways of helping children learn mathematics. This edition of *Helping Children Learn Mathematics* reflects the ever changing world of learning and teaching elementary school mathematics. You will experience many changes and have the opportunity to shape their direction.

Helping Children Learn Mathematics is built around three main themes:

- helping children make sense of mathematics
- incorporating practical experiences
- using research to guide teaching

Additionally, throughout the book you will also find connections and implications from the Common Core State Standards: Mathematics (CCSS-M).

Equity and diversity are key issues addressed throughout the book. Practical advice and suggestions about ways to help treat children equally and fairly are provided. Elementary teachers must strive to insure equity in their classrooms, and this becomes more challenging as the diversity within many classrooms grows.

This book is intended for those of you who are or who will be teachers of mathematics in elementary school. It is designed to help you help children develop understanding and proficiency with mathematics so they can solve problems. We hope our book will challenge your thinking and stimulate your interest in learning and teaching mathematics.

Helping Children Learn Mathematics consists of two main parts. The first part (Chapters 1–6) provides a basis for understanding the changing mathematics curriculum and how children learn mathematics. These chapters offer guidelines for planning, instructing, and evaluating that are aligned with the Mathematical Practices from the CCSS-M and include reference to the *Principles to Action: Ensuring Mathematical Success for All (NCTM, 2014)*. Particular attention is given to problem solving and assessment, both of which have profound implications for mathematics teaching. Their importance is reflected by the integration of problem solving and assessment throughout the book.

The second part (Chapters 7–18) discusses teaching strategies and techniques, as well as learning activities related to specific mathematical topics. The emphasis is on making mathematics make sense for our diverse population of students.

FEATURES OF THIS TEXT

Helping Children Learn Mathematics reflects current recommendations from professional associations, as well as recent research findings relevant to teaching mathematics. We have maintained the characteristics and features that have made this book a popular choice of instructors for many years. A useable text for instructors, it is also readable and understandable by students who are being introduced to teaching elementary mathematics. Yet, its depth also makes it appropriate for teachers to use as they continue to learn about teaching.

NEW Snapshots of a Lesson reflect video clips. These videos are new and drawn from the Teaching Channel. Each snapshot is aligned with the CCSS-M and is easily accessed online. Each chapter-opening snapshot is an excerpt from an actual mathematics lesson or mathematical situation. These videos demonstrate many effective classroom practices and provide a smooth, practical segue into the body of the chapter. You'll see the snapshots called out with this camera icon:

NEW Reflecting on the Snapshot Lesson provides questions to stimulate discussion about the Snapshot of a Lesson. Answering these questions will be a good preparation for licensure tests.

NEW Common Errors in Chapters 7–18 illustrate errors that children or teachers often make. Both research and experiences in working with children reveal frequent errors and misconceptions. We have identified errors that are often predictable and highlighted them in Common Error boxes. This feature is designed to stimulate your thinking as you strive to help children make sense of mathematics.

NEW Tech Connect. The Tech Connect boxes highlight a variety of technology resources, including apps for tablets and smartphones, and URLs that link to information aligned with the content being discussed.

NEW Learning Outcomes are identified early in each chapter and address major issues within each chapter. While these learning outcomes are not intended to be exhaustive, they do provide some key questions to guide reading and promote discussions.

Chapter on Fractions. Research has shown the importance of children having a conceptual understanding of fractions as well as fluency with computation. A chapter in this edition is

devoted entirely to common fractions. Decimals have been moved to the place value chapter.

In the Classroom activities are found throughout the text in special boxes. These activities provide a wealth of ideas and strategies for helping children explore, learn, and in some cases practice skills related to specific mathematical topics. Lesson plans for most of these activities can be easily found in the companion book, *Teaching Elementary Mathematics: A Resource for Field Experiences.*

Children's Literature. Literature can be a powerful ally in helping children learn. We have made a concerted effort to identify books that you can use effectively to stimulate children's interest in mathematics and to promote mathematics learning. We cite and discuss specific books at various points within the text, and we provide an updated annotated list of useful books in the **Book Nook** section at the end of each chapter (except Chapter 1).

Updated and Streamlined Research. Updated references to important research are found throughout the text. As a teacher, you are often called on to provide a rationale for curricular or instructional decisions, and we think you will find these references to relevant research useful; much of the research we cite has important implications for learning and teaching. Complete reference information for each citation is available on the book companion site at www.wiley.com/college/reys.

Things to Do sections found near the end of each chapter embody our active learning and teaching approach to mathematics. Divided into two parts, the Things to Do sections are designed to engage you in inquiring and thinking about mathematics—to offer you experiences and introduce you to investigations that will help you achieve the understanding and insight you need to be a successful teacher. The first part, "From What You've Read," relates directly to this book. The second part, "Going Beyond This Book," generally offers four types of activities: In the field, In your journal, With additional resources, and With technology. These include several activities linked to *Teaching Elementary Mathematics: A Resource for Field Experiences.*

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The Wiley **E-Text: Powered by VitalSource** gives students anytime, anywhere, access to the best math methods teaching content when and where they study: on their desktop, laptop, tablet, or smartphone. Students can search across content, highlight, and take notes that they can share with teachers and classmates.

This exciting new learning model brings textbook pages to life—no longer just a static e-book, the E-Text enriches the study experience with dynamic features:

- Pre- and Post-chapter quizzes
- *Teaching Channel* video introductions to each chapter

- Integrated *Teaching Elementary Mathematics: A Resource for Field Experiences* activities
- Access to student study tools, including Lecture PowerPoints and Blackline Masters

TEACHING AND LEARNING RESOURCES

Teaching Elementary Mathematics: A Resource for Field Experiences, 4th Edition

Featuring a wealth of materials to use in schools, this resource is designed for students to use when they participate in field experiences such as practicums, observations, and professional development school (PDS) experiences in K–8 classrooms. It is aligned with *Helping Children Learn Mathematics* and provides:

- **In the School** activities focus on observing and gathering information about the school and its resources. Resources for teacher and student interviews are included.
- **Helping Children Learn** activities include mini lessons for small groups or whole class, that include games, technology, and In the Activities in this *Resource* provide opportunities in collecting information about the school and school resources, observing and interviewing teachers and children, and doing mathematics with children in the form of mini-lessons, some involving games and technology. Pages from this *Resource* may be downloaded and printed for classroom use.

NEW Appendix and References

The Appendix and References are available on the book companion site at www.wiley.com/college/reys.

SUPPLEMENTS

The eleventh edition of *Helping Children Learn Mathematics* is accompanied by the following instructor and student supplements:

NEW Instructor's Manual

The Instructor's Manual is a useful resource for both veteran and new elementary methods instructors. It is available on the Wiley Web site at www.wiley.com/college/reys. Each chapter includes:

- Chapter overview, student objectives, and key vocabulary
- Supplemental lecture ideas, textbook extension ideas, and class and field activity suggestions
- Extensive resource list (both print and media)

Pre- & Post-Chapter Test

Objective questions are available online. These short quizzes may be helpful for focusing learning and reflecting on the content of each chapter. (More in-depth questions are available at the end of each chapter.)

Test Bank

Password protected in the instructor's section of www.wiley.com/college/reys, a test bank of over 500 items features both objective and open-ended questions with varying levels of complexity.

PowerPoint Slides

Our PowerPoint™ presentations contain a combination of key concepts allowing you to illustrate important topics.

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We also welcome any ideas you have for improving and strengthening this 11th edition of *Helping Children Learn Mathematics*.

Finally, we give a special thanks to **Joe Champion, Boise State University,** and **Erin Krupa, Montclair State University,** for their contributions to this book. They did a careful search of links and apps that would be useful for helping elementary school teachers. You will find many of their recommendations reflected in the Tech Connect boxes that appear throughout the book.

Reflecting on the many people who have helped shape this book, we say THANKS for your help and contributions.

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Contents

C H A P T E R 1

SCHOOL MATHEMATICS IN A CHANGING WORLD

"The schools ain't what they used to be and probably never were." —Will Rogers

SNAPSHOT OF A LESSON

KEY IDEAS

1. Setting expectations for students' responses.
2. Managing a classroom of young students.

BACKGROUND ▶

This snapshot, Silent Signals in a Math Classroom, is from the Teaching Channel and is available at https://www.teachingchannel.org/videos/student-silent-signals

OVERVIEW

Jen Saul is a third-grade teacher. In this brief part of a longer video, she shares ways she manages her students as they respond to questions in a math lesson. She has several hand silent signals that her students use to tell they are thinking, have an answer, or have a different answer, or to cheer for the student chosen to answer.

REFLECTING ON THE LESSON

1. What benefits and what drawbacks do you see in using these silent signals?
2. Think of yourself in elementary school. How would you have reacted to such signals?
3. If you were to adopt these signals in your classroom, how would you begin (would you introduce all of them, only one at a time, or modify them in some other way)?

CHAPTER LEARNING OUTCOMES

1. Contrast your view of mathematics to the five different views presented in this chapter.
2. Which of the three needs that influence what and how mathematics is taught is most important today? Why?
3. List the resources that are available to you to help children learn mathematics.

INTRODUCTION

As Will Rogers said, schools are not what they used to be, but our romantic view of the past is often flawed. Our young students today are as eager to learn as they ever were. We, as teachers, need ways to make that possible for all students. The teacher in the snapshot shared some of her techniques in fostering an environment that valued all students. Not only do we need to value all students, we need to value the subjects, in this case mathematics, we are teaching.

You have the opportunity to provide a positive and meaningful experience for those you help to learn mathematics. What is your vision of the mathematics you will be teaching? What is your vision of the classroom? Many of you will remember your experiences in elementary school. You remember memorizing multiplication tables, operating with fractions, and doing long division. Or, you may remember exploring patterns, doing geometry projects, and solving problems. Mathematics in elementary school may have been a positive experience for you, or it may have been filled with anxiety and frustration. Why were you learning mathematics and when would you use it? Teachers want students to learn mathematics and to learn that it is a useful subject. How can they make sure this happens?

This book is designed to expand your vision of teaching and learning mathematics and to help you help students learn mathematics. The book interweaves three main themes:

- *Theme 1. Best Practices and Research.* These best practices and research provide a basis for you to understand what mathematics children are expected to learn and how children learn mathematics.

- *Theme 2. Sense Making.* Mathematics should make sense to children. If children make sense of the mathematics they are learning, they can build on this understanding to learn more mathematics and use the mathematics to solve problems.

- *Theme 3. Practical Experiences.* Learning to teach mathematics requires experience. This theme is explicated by the many suggestions and ideas from teachers and our own experiences for you to use now and later in the classroom.

Learning to teach is a lifelong journey. During that journey, you will often ask questions such as these:

- What mathematical knowledge and understanding does each student bring to the class?
- What mathematics do students need to learn?
- How can I teach each unique child so that he or she will learn?
- How important is my own attitude toward mathematics?

Your answers to these questions will influence what you do when you are teaching. No matter what the age of the children you teach, we recommend three *general goals:*

- To help children make sense of specific mathematical content, including both procedures and concepts
- To help children learn how to apply mathematical ideas to solve problems
- To foster positive dispositions, such as persistence, flexibility, willingness to learn, and valuing mathematics

Developing ways to help you reach these three goals is considered in later chapters of this book. This first chapter focuses on what mathematics is and what determines the mathematics that is taught in schools. We also share where to turn for additional suggestions and help.

1. *Mathematics is a study of patterns and relationships.* Mathematics is filled with patterns and relationships providing threads that unify the curriculum. Children should come to see how one idea is like another. For example, children in first grade can see how one basic fact (say, $3 + 2 = 5$) is related to another basic fact (say, $5 - 3 = 2$). Older children can relate measuring to the nearest centimeter to rounding to the nearest hundred.

2. *Mathematics is a way of thinking.* Mathematics provides people with strategies for organizing, analyzing, and synthesizing information. Often symbolizing a real-life problem reduces it to a well-known mathematical procedure, making the problem easier to solve.

3. *Mathematics is an art, characterized by order and internal consistency.* Many children think of mathematics as a confusing set of discrete facts and skills that must be memorized. Children need guidance to recognize and appreciate the underlying orderliness and consistency to understand and use mathematics.

4. *Mathematics is a language that uses carefully defined terms and symbols.* Learning these terms and symbols enhances our ability to communicate about science, real-life situations, and mathematics itself. As with any language, you need to understand the meaning of these words and when it is appropriate to use them.

5. *Mathematics is a tool.* Mathematics has become an essential part of our world, both in everyday life and in the workplace. Children appreciate why they are learning mathematics if they know it is useful.

WHAT IS MATHEMATICS?

The view of mathematics in elementary school has changed from being mainly about numbers, especially computation, to a broader view. Numbers are essential, but algebra, geometry, measurement, and data analysis are also important.

Although we can consider mathematics as a collection of separate strands such as geometry and algebra, this may not be the best way of looking at it. It may be helpful to broaden your view of mathematics. Five views are presented to help you think of mathematics as being more than a collection of strands. Before reading these different views, take a minute to look at the quilt on the cover of this book. Do you see *patterns and relations* among the different parts of the quilt? Do you see the *thinking* underlying the quilt—the careful placement of the cloth pieces to make a whole? Do you see the quilt as *art*? What words or symbols would you use to describe the quilt? Do you see the quilt as a *tool*—something that would keep you warm? Similarly, you can look at mathematics in these ways:

WHAT DETERMINES THE MATHEMATICS BEING TAUGHT?

Mathematics plays a prominent role in the elementary school program. It is second only to reading in the amount of time devoted to it and in the amount of money spent for curricular materials. Its importance is reflected in the degree of concern about school mathematics voiced by parents, politicians, and other social groups.

HISTORICAL INFLUENCES

Three factors—the needs of the subject, the child, and the society—have influenced what mathematics is to be taught in schools. Many people think that "math is math" and never changes. A brief discussion of these three factors paints a different picture: mathematics is a subject that is ever changing.

Needs of the subject   The nature of mathematics helps determine what is taught and when it is taught in elementary grades. For example, number work begins with whole numbers, then

fractions and decimals. Length is studied before area. Such seemingly natural sequences are the result of long years of curricular evolution. This process has involved much analysis of what constitutes a progression from easy to difficult, based in part on what is deemed necessary at one level for the development of ideas at later levels. Once a curriculum is in place for a long time, however, people tend to consider it the only proper sequence. Thus, omitting a topic or changing the sequence of topics often involves a struggle for acceptance. However, research shows that all students do not always learn in the sequence that has been ingrained in our curriculum.

Sometimes the process of change is the result of an event, such as when the Soviet Union sent the first *Sputnik* into orbit. The shock of this evidence of another country's technological superiority sped curriculum change in the United States. The "new math" of the 1950s and 1960s was the result, and millions of dollars were channeled into mathematics and science education to strengthen school programs. Mathematicians became integrally involved. Because of their interests and the perceived weaknesses of previous curricula, they developed curricula based on the needs of the subject. The emphasis shifted from social usefulness to such unifying themes as the structure of mathematics, operations and their inverses, systems of notation, properties of numbers, and set language. New content was added at the elementary school level, and other topics were introduced at earlier grade levels.

Mathematics continues to change; new mathematics is created, and new uses of mathematics are discovered. As part of this change, technology has made some mathematics obsolete and has opened the door for other mathematics to be accessible to students. Think about all the mathematics you learned in elementary school. How much of this can be done on a simple calculator? What mathematics is now important because of the technology available today?

Needs of the child   The mathematics curriculum has been influenced by beliefs and knowledge about how children learn and, ultimately, about how they should be taught. Before the early years of the twentieth century, mathematics was taught to train "mental faculties" or provide "mental discipline." Struggling with mathematical procedures was thought to exercise the mind (like muscles are exercised), helping children's brains work more effectively. Around the turn of the twentieth century, "mental discipline" was replaced by *connectionism,* the belief that learning established bonds, or connections, between a stimulus and responses. This led teachers to the endless use of drills aimed at establishing important mathematical connections.

In the 1920s, the Progressive movement advocated *incidental learning,* reflecting the belief that children would learn as much arithmetic as they needed and would learn it better if it was not systematically taught. The teacher's role was to take advantage of situations when they occurred naturally as well as to create situations in which arithmetic would arise.

During the late 1920s, the Committee of Seven, a committee of school superintendents and principals from midwestern cities, surveyed pupils to find out when they mastered various topics (Washburne, 1931). Based on that survey, the committee recommended teaching mathematics topics according to students' mental age. For example, subtraction facts under 10 were to be taught to children with a mental age of 6 years 7 months and facts over 10 at 7 years 8 months; subtraction with borrowing or carrying was to be taught at 8 years 9 months. The recommendations of the Committee of Seven had a strong impact on the sequencing of the curriculum for years afterward.

Another change in thinking occurred in the mid-1930s, under the influence of *field theory,* or *Gestalt theory.* A 1954 article by William A. Brownell (2006), a prominent mathematics education researcher, showed the benefits of encouraging insight and the understanding of relationships, structures, patterns, interpretations, and principles. His research contributed to an increased focus on learning as a process that led to *meaning and understanding*. The value of drill was acknowledged, but it was given less importance than understanding; drill was no longer the major means of providing instruction.

The relative importance of drill and understanding is still debated today. In this debate, people often treat understanding and learning skills as if they are opposites, but this is not the case. Clearly, drill is necessary to build speed and accuracy and to make skills automatic. But equally clearly, you need to know *why* as well as *how*. Both skills and understanding must be developed, and they can be developed together.

Changes in the field of psychology have continued to affect education. During the second half of the twentieth century, educators came to understand that the developmental level of the child is a major factor in determining the sequence of the curriculum. Topics cannot be taught until children are developmentally ready to learn them. Or, from another point of view, topics must be taught in such a way that children at a given developmental level are ready to learn them.

Research has provided increasing evidence that children construct their own knowledge. In so doing, they make sense of the mathematics and feel that they can tackle new problems. Thus, helping children learn mathematics means being aware of how children have constructed mathematics from their experiences both in and out of school.

Read more about the influence of psychological theories, in the chapter by Lambdin and Walcott (2007).

Needs of society   The usefulness of mathematics in everyday life and in many vocations has also affected what is taught and when it is taught. In early America, mathematics was considered necessary primarily for clerks and bookkeepers. The curriculum was limited to counting; the simpler procedures for addition, subtraction, and multiplication; and some facts about measures and fractions. By the late nineteenth century, business and commerce had advanced to the point

that mathematics was considered important for everyone. The arithmetic curriculum expanded to include such topics as percentage, ratio and proportion, powers, roots, and series.

This emphasis on *social utility,* on teaching what was needed for use in occupations, continued into the twentieth century. One of the most vocal advocates of social utility was Guy Wilson. He and his students conducted numerous surveys to determine what arithmetic was actually used by carpenters, shopkeepers, and other workers. He believed that the dominating aim of the school mathematics program should be to teach those skills and only those skills.

In the 1950s, the outburst of public concern over the "space race" resulted in a wave of research and development in mathematics curricula. Much of this effort was focused on teaching the mathematically talented student. By the mid-1960s, however, concern was also being expressed for the disadvantaged student, as U.S. society renewed its commitment to equality of opportunity. With each of these changes, more and better mathematical achievement was promised.

In the 1970s, when it became apparent that the promise of greater achievement had not fully materialized, another swing in curriculum development occurred. Emphasis was again placed on the skills needed for success in the real world. The minimal competency movement stressed the basics. As embodied in sets of objectives and in tests, the basics were considered to be primarily addition, subtraction, multiplication, and division with whole numbers and fractions. Thus, the skills needed in colonial times were again being considered by many to be the sole necessities, even though children were now living in a world with calculators, computers, and other features of a much more technological society.

By the 1980s, it was acknowledged that no one knew exactly what skills were needed for the future but that everyone needed to be able to solve problems. The emphasis on problem solving matured through the last 20 years of the century to the point where problem solving was not seen as a separate topic but as a way to learn and to use mathematics (see Chapter 6).

Today, one need of our society is for a workforce that is competitive in the world. There is a call for school mathematics to ensure that students are ready for workforce training programs or college.

RECENT INFLUENCES

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM) In the late 1980s, the NCTM, the world's largest professional organization of teachers of mathematics, led the way in developing standards for school mathematics. It began a movement and all other curriculum professional organizations followed with standards for their subjects. The NCTM developed standards for curriculum and for evaluation, teaching, and assessment (NCTM, 1989, 1991, 1995). Because states and localities in the United States have the right to determine their own school policies, these standards were not prescriptive, but provided vision and direction for

schools. Many states and districts used these documents in preparing their guidelines.

In 2000, the NCTM updated the standards with the publication of *Principles and Standards for School Mathematics* (NCTM, 2000). The standards described the mathematical content and mathematical processes that should be taught in school mathematics. These influenced the development of the Common Core State Standards, the most recent guidelines. The principles represent fundamental beliefs about the characteristics of a high-quality, equitable mathematics program. The six NCTM principles represent fundamental beliefs about the characteristics of a high-quality, equitable mathematics program. We have included a brief discussion about each principle because of their relevance to today.

The Equity Principle   Excellence in mathematics education requires equity—high expectations and strong support for all students.

The Equity Principle states clearly that excellence in mathematics education means ensuring that all students learn mathematics. This vision can be realized only if each person involved in education firmly believes that all children can learn mathematics and that each child should be expected to do so. Every child must be given the opportunity to learn worthwhile mathematics. This means designing instructional programs that can encompass all the different interests, strengths, needs, cultures, and mathematical backgrounds of students. Plenty of evidence supports the idea that all students can learn mathematics. High-quality instructional programs are needed that let well-prepared teachers and other school personnel respond to students' varied strengths and needs.

Our schools are characterized by diversity—students from many different cultures and languages and from many different economic and home backgrounds, with different strengths, ways of learning mathematics, and past experiences with mathematics. Equity *does* mean that all children must learn worthwhile mathematics, but it *does not* mean that all should have the same instruction. In fact, it means that children can reach the high expectations set for them only if we meet the individual needs of each child. Your repertoire of ways to reach children will grow as you teach and learn. At this point, you can begin by challenging the popular belief that only some children can learn mathematics. This is an important first step in becoming a teacher who can help every child learn mathematics.

The Curriculum Principle   A curriculum must be coherent, focused on important mathematics, and well articulated across the grades.

- *Coherent.* A curriculum that fits mathematical ideas together in a meaningful way
- *Focused.* A curriculum that focuses on the important mathematics topics and ideas at each grade, not on every possible topic
- *Well articulated.* A curriculum that builds on previous learning and grows across the grades

No one knows exactly what mathematics will be needed as the twenty-first century progresses, but it is clear that students will need to know how to reason mathematically and how to apply mathematical thinking to a wide range of situations. How you view mathematics will determine how you view teaching mathematics. If you view mathematics as a collection of facts to learn and procedures to practice, then you will teach that to your students. If you view mathematics as a logical body of knowledge, you will design your program to guide children in making sense of mathematics. Chapters 7–18 look at specific content and ways to help you help children.

The Learning Principle   Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

What it means to learn mathematics has changed a great deal over the past century. Currently, the phrase mathematical proficiency is used to describe what it means to learn mathematics successfully. Ideas about developing mathematical proficiency are considered in more depth in Chapter 2.

In a changing world, learning mathematics with understanding is essential in order to meet this goal of mathematical proficiency. Research has shown that if children are able to make sense of the mathematics they are learning, they can build on this understanding to learn more mathematics and use that mathematics to solve problems in order to become mathematically proficient.

The Teaching Principle   Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

To teach mathematics effectively, teachers must know more than just mathematics. They need to know their students as learners, and they must adjust their pedagogical strategies in response to students' varying experiences. Teachers must design lessons that reveal to them what students already know, that reveal students' misunderstandings, and that guide students to construct more complex understandings of mathematics. Teachers must create challenging and supportive classroom learning environments that help children make sense of mathematics. Teachers must also encourage students to think, question, solve problems, and discuss their ideas. Chapter 3 initiates the discussion of teaching, and succeeding chapters focus on ways to teach and on useful types of activities.

The Assessment Principle   Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

People often think of assessment as testing to see what students have learned. The Assessment Principle presents a much broader view of assessment. Helping all students learn mathematics requires that assessment be an integral part of the instructional program. But assessment should not be something that is done to students; rather, a mathematics program must include assessments that are done for students, to guide and enhance their learning. The Assessment Principle is considered in more detail in Chapter 4.

The Technology Principle   Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

You will teach at a time when technology dominates activities both in and out of school. The Technology Principle acknowledges that technology will continue to be important in teaching and learning mathematics, as long as it enhances what is being learned and how it is being taught. As you teach your classes, you should keep asking three questions:

1. How can I help children use technology appropriately?
2. What mathematics do children need in order to use technology wisely?
3. What mathematics is no longer necessary because of technology?

Some parents continue to be concerned about the use of calculators in learning mathematics in elementary schools. A meta-analysis of 54 research studies on the use of and attitudes toward calculators (Ellington, 2003) suggests that using calculators does not hinder the development of mathematical skills and that students who used calculators had better attitudes toward mathematics than those who did not. Of course, children need to learn to use calculators appropriately, as they do any other tool (see Chapter 10).

COMMON CORE STATE STANDARDS FOR MATHEMATICS (CCSSM)   The most recent effort, the Common Core State Standards Initiative (CCSSI) to set standards and associated assessments, was led by the National Governors Association for Best Practices and the Council of Chief State School Officers. These standards for reading and for mathematics were informed by the NCTM standards, state standards, and standards from around the world. The *Common Core State Standards for Mathematics* defines the mathematical knowledge and skills students should obtain from kindergarten through high school.

This is an attempt to have a common understanding across the United States of what students are expected to learn at each grade level. The states that choose to adopt these standards (and most have) will have a period of time to align their standards with these. Common assessments, both summative and formative, are being developed by centers (Smarter Balanced and Partners for Assessment of Readiness for College and Careers) with consortia of the states that will use these assessments beginning in 2014. These assessments will be available electronically with the expectation that students will be familiar with technology. The technology can allow for many innovations in testing.

As with any change, there are many questions raised when states, schools, and teachers begin to implement these standards. Helpful and thoughtful suggestions can be found in *Curriculum Issues in an Era of Common Core State Standards for Mathematics* (Hirsch, Lappan, & Reys, 2012).

CCSSM Notation

Mathematics Standards
Example: 4.OA.B.3

Grade 4
Domain OA (Operations & Algebraic Thinking)
Cluster B (Gain familiarity with factors and multiples)
Objective 3 (more specific statement)

Mathematics Practices (MP)
Example: MP2

The practices are the same for Kindergarten through Grade 12, so MP2 represents the second (2) practice.

Figure 1-1 *Notation used with CCSSM.* (Source: Copyright 2010 by the National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.)

The NCTM (2014) has published a document entitled, *Principles to Action: Ensuring Mathematical Success for All.* This document builds on the six principles discussed in the previous section adding a seventh principle on professionalism. It describes the barriers that must be overcome in order for a successful implementation of CCSSM.

In this text, we have included the relevant standards in the content-focused chapters. It is important that you understand these standards and begin to use them in your preparation for teaching. The CCSSM also includes a list of eight mathematical practices that are fundamental to the vision and implementation. These are fully discussed in Chapter 4.

Abbreviations are commonly used in referring to a specific standard or mathematical practice; Figure 1-1 provides a guide to reading these abbreviations.

Federal legislation   A serious effort to hold schools accountable for student learning began with the new century. Accountability was one of the pillars of the No Child Left Behind Act (NCLB) of 2001, which required states to show that each school is making yearly progress in academic subjects and that all students are becoming mathematically proficient. However, it was left to the states to decide what is meant by mathematically proficient. The No Child Left Behind legislation, which calls for annual testing in reading and mathematics each year in grades 3–8 and once in grades 10–12, provides "an example of federal funding that is linked, at least in part, to the results of standardized achievement tests at the state level. Every state has now instituted some sort of statewide testing program, partly in response to this legislation" (Wilson, 2007, p. 1099). The plan now is to base many of these testing programs on the CCSSM.

Along with its focus on accountability, the NCLB called for supporting supplementary services and professional development for teachers. Unfortunately, the majority of the available funds have gone into testing for accountability. Poorly performing schools are still struggling and are still in need of well-prepared teachers to meet the challenge of helping all students become mathematically proficient.

High stakes assessments   Today's society is focused on assessments in the form of tests that are used to compare students' performance across schools, states, and nations. These summative assessments are primarily designed to document what students know and are able to do. Tests are sometimes given to make decisions about students—which class to place students in, what grade to record on their report card, or whether to promote them to the next grade. When assessments have serious consequences such as these, we call them "high-stakes assessments." In the situations just described, high-stakes decisions about the students themselves are being made as a result of the assessments.

It is not just the individual students, however, who are affected by the consequences of high-stakes assessments. Tests may be administered in order to document the achievement of a group of students or to compare one group of students with another. Individual teachers, or schools, or districts, may be held accountable for their students' test scores, taken as a group. Teachers and schools may be rated or ranked according to the results of such tests. Many schools, districts, and states now use student test results to make funding decisions, to help determine teacher's pay, or even to decide who will keep their jobs.

Teachers feel pressure to "teach to the test." This is not all bad if the tests actually measure what is important and allow students access to show what they really know and can do. However, many of the tests focus only on lower level skills. While mathematics proficiency requires a level of skill, say, in computation, it requires much more. If children have the opportunity to learn the content on the tests in a manner that makes sense to them, they will do well on such tests. Teach to the standards that are set for your school district and community.

WHERE CAN YOU TURN?

There are many places you can turn to develop your knowledge of mathematics and of mathematics learning and teaching. In this section, we discuss a few of the resources that we reference throughout this book and that you can use now and in teaching.

NATIONAL GUIDELINES FOR SCHOOL MATHEMATICS

We have discussed the role of *The Common Core State Standards for School Mathematics* (CCSSI, 2010) previously in this chapter. The relevant standards are discussed in the remaining chapters, but you may access the full document as shown in Tech Connect 1.1.

> ### *Tech Connect 1.1*
>
> You can access *The Common Core State Standards for Mathematics* from http://www.corestandards.org/ or from this book's Web site.
>
> **www.wiley.com/college/reys**

We also frequently refer to the *Principles and Standards for School Mathematics* (NCTM, 2000). In this chapter, we have briefly discussed the principles. The five process standards along with CCSSM mathematical practices are included in Chapter 5. The five content standards provide structure to Chapters 7–18 which are elucidated through the CCSSM standards.

STATE AND LOCAL GUIDELINES

Almost every state has a document with standards, guidelines, or frameworks for school mathematics. Although there is much commonality among states, a study (Reys, 2006) of the grade-level expectations of states found a wide variety in the specificity and in the grade in which a skill was targeted.

For example, expectation of mastery of basic addition facts ranges from grade 1 to grade 3. Of the 38 states that specify the grade level, 21% indicate grade 1, 74% indicate grade 2, and 5% indicate grade 3 (Reys, 2006). Consequently, textbooks often contain material not appropriate for your state's guidelines. One reason for developing CCSSM is the vast difference among states. Ask yourself if it is fair for children from one state to have less of an opportunity to learn mathematics than children from another state.

You need to become familiar with your state document and use it to plan when you are teaching. Links to state mathematics curriculum frameworks are available from the Center for the Study of Mathematics Curriculum (http://www.mathcurriculumcenter.org/states.php). Many localities have their own versions of the state's document. They often expect more of students than the state documents and connect the expectations to their mathematics program.

RESEARCH

Research is referenced throughout this book, not only to acquaint you with research in mathematics education, but also to illustrate or support discussions in the text. There is a substantial body of research in mathematics education, both about children's learning and about teaching. We often use the following two sources: *Second Handbook for Research on Mathematics*
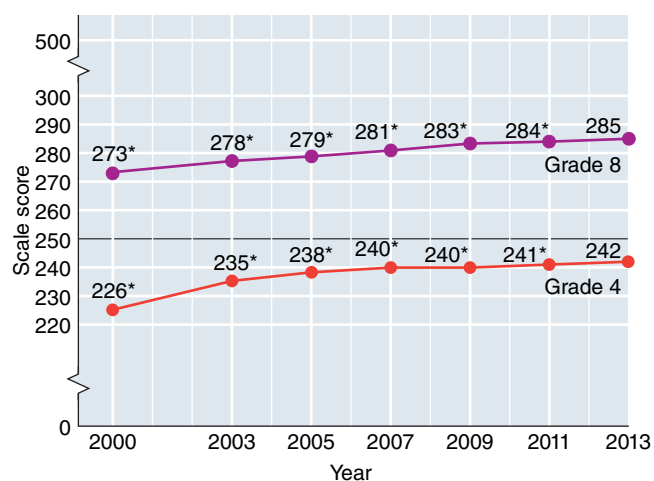
Figure 1-2   *Trends in mathematics performance.* (Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics.)

Learning and Teaching (Lester, 2007), and *Teaching and Learning Mathematics: Translating Research for Elementary School Teachers* (Lambdin, 2010). The second reference is written especially for easy and practical access to research.

The National Assessment of Educational Progress (NAEP, pronounced "nape") is the nation's measure of students' achievement and trends in achievement in the academic subjects. With the passage of the NCLB in 2001, NAEP has become more prominent. The present framework for NAEP elementary mathematics is closely aligned with the NCTM's standards. The NAEP assessment contains a variety of types of items (multiple choice, short response, and open-ended response). The mathematics assessment is given to a sample of students at grades 4 and 8 in every state. Results are reported by states as well as by race, gender, and socioeconomic status. Figure 1-2 shows the overall national results for the years 1990 through 2011. Note that both grades showed significant improvement over this period.

> ### *Tech Connect 1.2*
>
> More information about NAEP, including sample items, may be found at http://nces.ed.gov/nationsreport-card/mathematics or from this book's Web site.
>
> **www.wiley.com/college/reys**

Research reports appear in many journals. We often use articles from the research journal of the NCTM, *Journal for Research in Mathematics Education* (*JRME*), to guide our recommendations in this book. *JRME* articles often lead directly to the classroom ideas and recommendations found in other NCTM journals, such as *Teaching Children Mathematics*. The Research Clips and Briefs found on NCTM's Web site give snippets of recent and relevant research.

Tech Connect 1.3

The NCTM provides a wealth of research and references. You can access this from http://www.nctm.org (either as a member of NCTM or via the 120-day free access) or from this book's Web site.

www.wiley.com/college/reys

CULTURAL AND INTERNATIONAL RESOURCES

The Trends in Mathematics and Science Study (TIMSS), an international study, is given at grades 4 and 8. The results of a recent TIMSS study, in 2011, showed that U.S. fourth-grade students were above the international average. The average score for fourth-grade students in 8 countries were higher, 6 countries were the same, and 42 countries were lower. Eighth-grade U.S. students were also above the international average. Performance varies greatly within the United States and is closely linked to economic status.

The TIMSS study also collects information about curriculum, teaching, and teachers. These results suggest that U.S. educators can learn much from analyzing how other countries teach mathematics.

Many of the other chapters in this book suggest resources that will help you understand how mathematics is taught and learned in other cultures and that show you ways to use culturally oriented activities as you strive to reach each child.

Tech Connect 1.4

You can find more information about TIMSS, including sample items, at http://www.nces.ed.gov/timss or from this book's Web site.

www.wiley.com/college/reys

TEXT BOOKS AND OTHER MATERIALS

There are many different types of textbooks. Some provide a lot of drill and practice but offer little help in developing understanding and using mathematics. Others may do the opposite, providing a great deal of help in developing students' understanding but falling short on practice of necessary skills. As you gain experience, you will be better able to judge the quality of a textbook and better able to depart from it as appropriate. If you have not had much experience, you may want to stick with the textbook until you become more comfortable with teaching. But be sure always to ask why you need to teach a given lesson. Does it help children develop the mathematics they need? Does it help children make sense of the mathematics?

Today's textbooks provide supplementary materials such as assessments, problems to solve, and extra practice. Teachers' manuals also provide a wealth of materials and teaching suggestions.

ELECTRONIC MATERIALS

The kind and amount of materials readily available are increasing every day. The Web provides immediate access to lesson plans, help with the mathematics itself, assessment items, and information that can be used in teaching mathematics. Additionally, many sites have videos of mathematics classes. Each chapter of this book begins with a Snapshot of a Lesson. These snapshots are excerpts from videos that illustrate teaching related to the topic of the chapter and to CCSSM. Watching the entire video will give you a better feel for interactions of the teacher and students.

The Tech Connects included in each chapter recommend sites that are worth investigating. A simple search of the Internet for information about mathematics for elementary students will lead you to many other Web sites. Availability on the Web is no guarantee of quality. A prospective teacher cited an activity from a Web site in which young children found the capacity of their mouths by filling them with marshmallows! The mathematics is questionable because the unit of measurement could change as the marshmallows are squashed. More importantly, children could easily choke on a mouthful of marshmallows. Teaching involves making good judgments on many fronts.

Tech Connect 1.5

The Math Forum Web site includes a mathematics library, a discussion board for teachers, lesson plans, activities created and submitted by teachers, Problems of the Week for students, and answers to math problems from Ask Dr. Math. You can access this Web site at http://www.mathforum.org/ or from this book's Web site.

www.wiley.com/college/reys

There are many electronic sites related to CCSSM. In particular, you should become familiar with the sites of the consortiums producing the assessments. Partners for Assessment of Readiness for College and Careers (PARCC) and Smarter Balanced are both working with many states to design assessments that will be delivered electronically. Think about the possibilities of electronic assessments to accommodate the diverse needs of students.

Tech Connect 1.6

PARCC and Smarter Balanced sites include a list of states in their consortia, their philosophy, and sample questions for assessing the CCSSM standards in grades 3–8. You can access these sites at http://www.parcconline.org and at http://www.smarterbalanced.org or from this book's Web site.

www.wiley.com/college/reys

PROFESSIONAL ORGANIZATIONS

Professional association with others and the support you can find from being a part of a professional organization will enhance your teaching career. Journals, conferences, and other materials of a professional organization are often available through schools.

The NCTM (www.nctm.org) offers many publications, including a journal for elementary teachers of mathematics *(Teaching Children Mathematics)* and one for middle school teachers *(Mathematics Teaching in the Middle School).* You will find many references in this book to these journals. The NCTM also sponsors conferences, e-workshops, and other support activities. There are many affiliated state and local groups of NCTM that offer publications and programs.

PROFESSIONAL DEVELOPMENT

Start taking advantage of professional development opportunities. Some of these will be formal, such as workshops, college courses, and conferences; others will include informal study groups. Your school, district, or state will provide some opportunities for you. Others will be commercially sponsored or sponsored by a professional organization. An increasing number of opportunities on the Web are designed so you can participate as your schedule permits.

Often, schools or districts have funds set aside for professional development that are available on request. Some districts have teachers design their own professional development plans and support them in carrying out those plans.

OTHER TEACHERS

Teachers learn from each other. You will learn from your school experiences, but do not let learning stop there. Look for schools where sharing ideas about helping students learn mathematics and sharing teaching tips and materials are the norm. Look for teachers in other schools, either near you or far away but connected electronically, who are willing to discuss and to share. A good teacher who is willing to work with you is an invaluable resource.

YOURSELF

One of the most important resources is you. As you prepare for teaching mathematics, be sure to think about the broader context of your work and carefully consider these three challenges:

- *Examine your own disposition toward mathematics and your beliefs about who can learn mathematics.* Be ready to question your beliefs, to evaluate proposed changes, and to make a difference in helping children learn mathematics.

- *Take seriously the title of this book. Teaching means helping students learn, not merely giving out information.* As you begin working with children, stop and listen to them, individually and collectively. Reflect on what you are hearing, and learn with and from the children.

- *Realize that doing mathematics and teaching mathematics are different.* Teaching mathematics requires a depth of understanding about mathematics, about students, about schools, about curriculum, and about pedagogy. If you come to this realization and actively seek knowledge and experiences that integrate these areas, you are well on your way to becoming a good teacher.

WHAT IS YOUR ROLE NOW?

Teaching mathematics in a changing world means that the curriculum and instruction must change to reflect the needs of the subject, the child, and society. In this chapter, you have been challenged to consider your view of mathematics as a subject. You have seen a glimpse of the changes through the past as well as recommendations for teaching mathematics in the twenty-first century. The six principles put forth by the NCTM underpin many of the recommendations for today. Resources have been identified to support your study throughout the rest of this book and, more important, as you teach. The challenge is to keep an open mind and continue your own learning about teaching children mathematics. Prepare to help your students make sense of mathematics.

Things to Do: *From What You've Read*

1. What are the three general goals mentioned in the introduction? Which do you think is the most important? Explain why?

2. What is the purpose of CCSSM? What does it include?

3. Which of the resources in Where Can You Turn have you already used? Which ones do you think will be most helpful to you? Why?

4. Explain in your own words the six principles that underpin *Principles and Standards for School Mathematics.* Which is the most important to you? Why?

Things to Do: *Going Beyond This Book*

In the Field

1. *Mathematics in the School.* As you observe in a school, look for signs of the role that mathematics plays in that school. Does it differ from class to class?

2. *Equity: Interview the Teacher.* What does the statement "all children can learn mathematics" mean to you? Interview a teacher and compare his or her answer to yours.